C4 Vectors

1. June 2010 qu.6

Lines l_1 and l_2 have vector equations $\mathbf{r} = \mathbf{j} + \mathbf{k} + t(2\mathbf{i} + a\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 3\mathbf{i} - \mathbf{k} + s(2\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$ respectively, where t and s are parameters and a is a constant.

(i) Given that l_1 and l_2 are perpendicular, find the value of a. [3]

(ii) Given instead that l_1 and l_2 intersect, find

(a) the value of a, [4]

(b) the angle between the lines. [3]

2. Jan 2010 qu.2

Points A, B and C have position vectors $-5\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$ respectively, where p is a constant.

(i) Given that angle $ABC = 90°$, find the value of p. [4]

(ii) Given instead that ABC is a straight line, find the value of p. [2]

3. Jan 2010 qu.9

The equation of a straight line l is $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. O is the origin.

(i) The point P on l is given by $t = 1$. Calculate the acute angle between OP and l. [4]

(ii) Find the position vector of the point Q on l such that OQ is perpendicular to l. [4]

(iii) Find the length of OQ. [2]

4. June 2009 qu.7

(i) The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $4\mathbf{i} + \mathbf{k}$ and to the vector $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.

Find the values of b and c, and show that u is a unit vector. [6]

(ii) Calculate, to the nearest degree, the angle between the vectors $4\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. [3]

5. Jan 2009 qu.7

(i) Show that the straight line with equation $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ meets the line passing through (9, 7, 5) and (7, 8, 2), and find the point of intersection of these lines. [6]

(ii) Find the acute angle between these lines. [4]

6. June 2008 qu.4

Relative to an origin O, the points A and B have position vectors $3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ respectively.

(i) Find a vector equation of the line passing through A and B. [2]

(ii) Find the position vector of the point P on AB such that OP is perpendicular to AB. [5]

7. June 2008 qu.6

Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}.$$

(i) Show that the lines intersect. [4]

(ii) Find the angle between the lines. [4]

8. Jan 2008 qu.1

Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. [4]

9. Jan 2008 qu.5

The vector equations of two lines are

$$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}) + s(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = (2\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) + t(2\mathbf{i} - \mathbf{j} - 5\mathbf{k}).$$

Prove that the two lines are

(i) perpendicular, [3]

(ii) skew. [5]

10. June 2007 qu.9

Lines L_1, L_2 and L_3 have vector equations

$$L_1: \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$
$$L_2: \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$
$$L_3: \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

(i) Calculate the acute angle between L_1 and L_2. [4]

(ii) Given that L_1 and L_3 are parallel, find the value of c. [2]

(iii) Given instead that L_2 and L_3 intersect, find the value of c. [5]

11. Jan 2007 qu.3

The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to an origin O, where $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -7\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$.

(i) Find the length of AB. [3]

(ii) Use a scalar product to find angle OAB. [3]

12. Jan 2007 qu.10

The position vectors of the points P and Q with respect to an origin O are $5\mathbf{i} + 2\mathbf{j} - 9\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ respectively.

(i) Find a vector equation for the line PQ. [2]

The position vector of the point T is $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ. [4]

It is given that OT intersects PQ.

(iii) Find the position vector of the point of intersection of OT and PQ. [3]

(iv) Hence find the perpendicular distance from O to PQ, giving your answer in an exact form. [2]

13. June 2006 qu.4

The position vectors of three points A, B and C relative to an origin O are given respectively by

$$\overrightarrow{OA} = 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$
$$\overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$$
$$\text{and} \quad \overrightarrow{OC} = 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.$$

(i) Find the angle between AB and AC. [6]

(ii) Find the area of triangle ABC. [2]

14. June 2006 qu.7

Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k}),$$

where a is a constant.

(i) Given that the lines are skew, find the value that a cannot take. [6]

(ii) Given instead that the lines intersect, find the point of intersection. [2]

15. Jan 2006 qu.9

Two lines have vector equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + t\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -2 \\ a \\ -2 \end{pmatrix} + s\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix},$$

where a is *a* constant.

(i) Calculate the acute angle between the lines. [5]

(ii) Given that these two lines intersect, find *a* and the point of intersection. [8]

16. June 2005 qu.3

The line L_1 passes through the points (2, –3, 1) and (–1, –2, –4). The line L_2 passes through the point (3, 2, –9) and is parallel to the vector $4\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

(i) Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2]

(ii) Prove that L_1 and L_2 are skew. [5]

17. June 2005 qu.5

ABCD is a parallelogram. The position vectors of A, *B* and *C* are given respectively by

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \qquad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j}, \qquad \mathbf{c} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}.$$

(i) Find the position vector of *D*. [3]

(ii) Determine, to the nearest degree, the angle *ABC*. [4]